ENGINEERING PHYSICS

CONTENTS

UNIT-1

DIMENSIONS AND VECTORS

DIMENSION & DIMENSIONAL FORMULA OF PHYSICAL QUANTITIES-

Dimensions: **Dimensions of a physical quantity are, the powers to which the fundamental units are raised to get one unit of the physical quantity**.

The fundamental quantities are expressed with following symbols while writing dimensional formulas of derived physical quantities.

- **Mass →[M]**
- **Length→[L]**
- **Time→[T]**
- **Electric current →[I]**
- **Thermodynamic temperature →[K]**
- **Intensity of light →[cd]**
- **Quantity of matter →[mol]**

Dimensional Formula : **Dimensional formula of a derived physical quantity is the "expression showing powers to which different fundamental units are raised".**

Ex : Dimensional formula of Force F →[$M^1 L^1 T^{-2}$]

Dimensional equation: **When the dimensional formula of a physical quantity is expressed in the form of an equation by writing the physical quantity on the left hand side and the dimensional formula on the right hand side, then the resultant equation is called Dimensional equation.**

Ex: Dimensional equation of Energy is E = [$M^1 L^2 T^{-2}$] .

Derivation of Dimensional formula of a physical quantity:-

The dimensional formula of any physical quantity can be derived in two ways.

i) **Using the formula of the physical quantity** :

Ex: let us derive dimensional formula of Force .

Force F→ ma ;

substituting the dimensional formula of mass m →[M] ;

acceleration →[LT^{-2}]

We get F → [M][LT^{-2}]; **F →[$M^1L^1T^{-2}$]** .

ii) **Using the units of the derived physical quantity**.

Ex: let us derive the dimensional formula of momentum.

Momentum (p) → kg – mt – sec^{-1}

kg is unit of mass → [M] ;

metre (mt) is unit of length → [L] ;

sec is the unit of time →[T]

Substituting these dimensional formulas in above equation we get

p →[$M^1L^1T^{-1}$].

- **Quantities having no units, can not possess dimensions**:

The following physical quantities neither possess units nor dimensions.

Trigonometric ratios, logarithmic functions, exponential functions, coefficient of friction, strain, poisson's ratio, specific gravity, refractive index, Relative permittivity, Relative permeability.

• **Quantities having units, but no dimensions** :
The following physical quantities possess units but they do not possess any dimensions.

Plane angle, angular displacement, solid angle.

• **Quantities having both units & dimensions** :
The following quantities are examples of such quantities.

Area, Volume,Density, Speed, Velocity, Acceleration, Force, Energy etc.

Physical Constants : These are two types

i) Dimension less constants (value of these constants will be same in all systems of units):

Numbers, pi, exponential functions are dimension less constants.

ii) Dimensional constants (value of these constants will be different in different systems of units):

Universal gravitational constant (G),plank's constant (h), Boltzmann's constant (k), Universal gas constant (R), Permittivity of free space(ϵ_0) , Permeability of free space (μ_0),Velocity of light (c).

Principle of Homogeneity of dimensions:

The term on both sides of a dimensional equation should have same dimensions. This is called principle of Homogeneity of dimensions.

(or) Every term on both sides of a dimensional equation should have same dimensions. This is called principle of homogeneity of dimensions.

Uses of Dimensional equations :

Dimensional equations are used

i) to convert units from one system to another,
ii) to check the correctness of the dimensional equations
iii) to derive the expressions connecting different physical quantities.

CHECKING THE CORRECTNESS OF PHYSICAL EQUATIONS:

According to the Principle of Homogeneity, if the dimensions of each term on both the sides of equation are same, then the physical quantity will be correct.

The correctness of a physical quantity can be determined by applying dimensions of each quantity

Example 1

To check the correctness of $v = u + at$, using dimensions

Dimensional formula of final velocity $v = [LT^{-1}]$

Dimensional formula of initial velocity $u = [LT^{-1}]$

Dimensional formula of acceleration x time, $at = [LT^{-2} \times T] = [LT^{-1}]$

Dimensions on both sides of each term is the same. Hence, the equation is dimensionally correct.

Example 2

Consider one of the equations of constant acceleration,

$$s = ut + 1/2\ at^2.$$

The equation contains three terms: s, ut and $1/2at^2$.

All three terms must have the same dimensions.

- s: displacement = a unit of length, L
- ut: velocity x time = $LT^{-1} \times T = L$
- $1/2at^2$ = acceleration x time = $LT^{-2} \times T^2 = L$

All three terms have units of length and hence this equation is dimensionally correct.

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RESOLUTION OF VECTORS:

Definition:-

- **The process of splitting a vector into various parts or components is called "RESOLUTION OF VECTOR"**
- These parts of a vector may act in different directions and are called "components of vector".

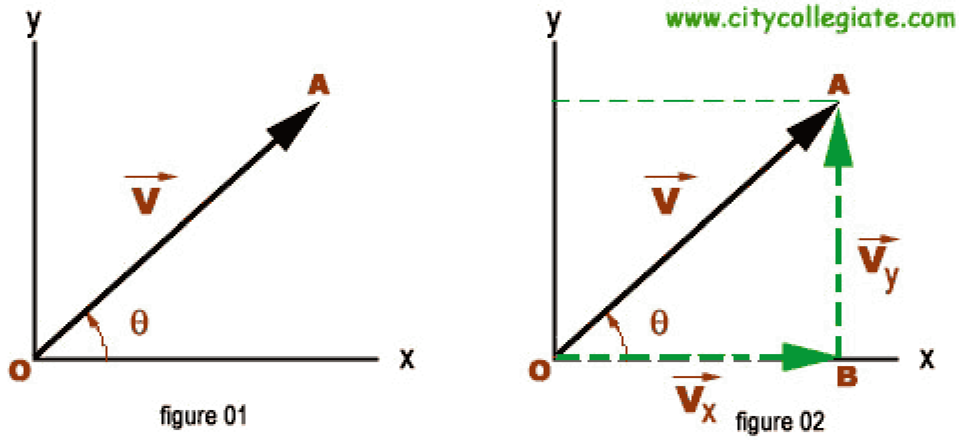
A vector can be resolved into a number of components .Generally there are three components of vector .
Component along X-axis called x-component
Component along Y-axis called Y-component
Component along Z-axis called Z-component

Let us consider only two components x-component & Y-component which are perpendicular to each other. **These components are called rectangular components of vector**.

Method of resolving a vector into its rectangular components:-
Consider a vector $\vec{v}$ acting at a point making an angle □□□with positive X-axis. Vector □$\bar{v}$ is
represented by a line OA. From point A draw a perpendicular AB on X-axis. Suppose OB and BA
represents two vectors. Vector $\overrightarrow{OA}$ is parallel to X-axis and vector $\overrightarrow{BA}$ is parallel to Y-axis. Magnitude
of these vectors are V_x and V_y respectively. By the method of head to tail we notice that the sum of these vectors is equal to
vector $\vec{v}$.Thus V_x and V_y are the rectangular components of vector $\vec{v}$.

V_x = Horizontal component of $\vec{V}$.

V_y = Vertical component of $\vec{V}$.



MAGNITUDE OF HORIZONTAL COMPONENT:

Consider right angled triangle ????

$$\cos\theta = \frac{\overline{OB}}{\overline{OA}}$$

$$\overline{OB} = \overline{OA}\cos\theta$$

$$\boxed{V_x = V\cos\theta}$$

MAGNITUDE OF VERTICAL COMPONENT:

Consider right angled triangle ????

$$\sin\theta = \frac{\overline{AB}}{\overline{OA}}$$

$$\overline{AB} = \overline{OA}\sin\theta$$

$$\boxed{V_y = V\sin\theta}$$

DOT PRODUCT AND CROSS PRODUCT OF VECTORS :-

DOT PRODUCT:

The Dot Product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}.\vec{B}$

$$\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\| \, \|\mathbf{B}\| \cos\theta,$$

Where θ is the angle between **A** and **B**.

In particular, if **A** and **B** are orthogonal, then the angle between them is 90° and

$$\mathbf{A} \cdot \mathbf{B} = 0.$$

At the other extreme, if they are codirectional, then the angle between them is 0° and

$$\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\| \, \|\mathbf{B}\|$$

This implies that the dot product of a vector **A** by itself is

$$\mathbf{A} \cdot \mathbf{A} = \|\mathbf{A}\|^2,$$

which gives

$$\|\mathbf{A}\| = \sqrt{\mathbf{A} \cdot \mathbf{A}},$$

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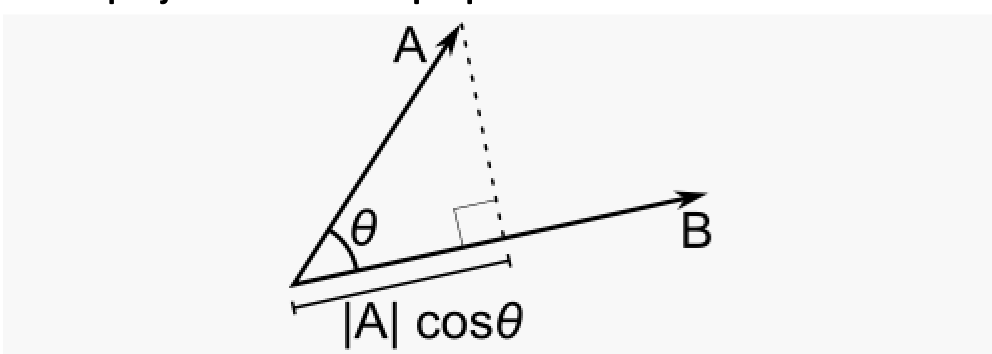
Scalar projection and first properties



The scalar projection (or scalar component) of a vector **A** in the direction of vector **B** is given by

$$A_B = \|\mathbf{A}\| \cos\theta$$

Where θ is the angle between **A** and **B**.

In terms of the geometric definition of the dot product, this can be rewritten

$$A_B = \mathbf{A} \cdot \widehat{\mathbf{B}}$$

Where $\widehat{\mathbf{B}} = \mathbf{B}/\|\mathbf{B}\|$ is the unit vector in the direction of **B**.

The dot product is thus characterized geometrically by

$$\mathbf{A} \cdot \mathbf{B} = A_B\|\mathbf{B}\| = B_A\|\mathbf{A}\|.$$

The dot product, defined in this manner, is homogeneous under scaling in each variable, meaning that for any scalar α,

$$(\alpha\mathbf{A}) \cdot \mathbf{B} = \alpha(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \cdot (\alpha\mathbf{B}).$$

The dot product also satisfies a distributive law, meaning that

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}.$$

$\mathbf{A} \cdot \mathbf{A}$ is never negative and is zero if and only if $\mathbf{A} = \mathbf{0}$.

Properties of scalar product of vectors.

1. **Commutative:**

   $$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}.$$

   which follows from the definition (ϑ is the angle between **a** and **b**):

   $$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta = \|\mathbf{b}\|\|\mathbf{a}\|\cos\theta = \mathbf{b} \cdot \mathbf{a}$$

2. **Distributive over vector addition:**

   $$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}.$$

3. **Bilinear**:

   $$\mathbf{a} \cdot (r\mathbf{b} + \mathbf{c}) = r(\mathbf{a} \cdot \mathbf{b}) + (\mathbf{a} \cdot \mathbf{c}).$$

4. **Scalar multiplication:**

   $$(c_1\mathbf{a}) \cdot (c_2\mathbf{b}) = c_1c_2(\mathbf{a} \cdot \mathbf{b})$$

5. **Orthogonal:**

   Two non-zero vectors **a** and **b** are *orthogonal* if and only if **a** · **b** = 0.

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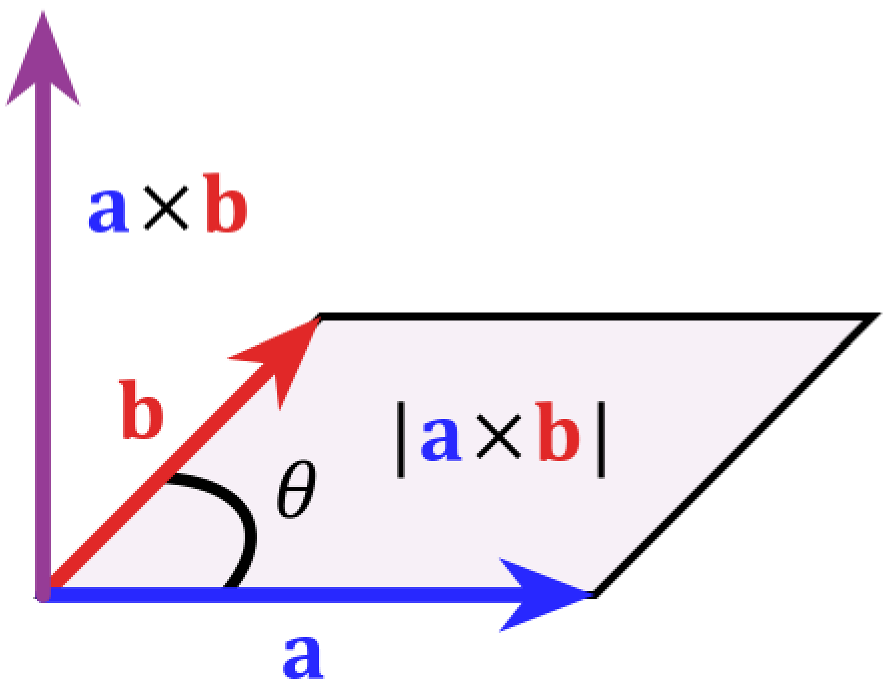
CROSS PRODUCT :

The cross product of two vectors **a** and **b** is defined only in three-dimensional space and is denoted by **a** × **b**.

The cross product **a** × **b** is defined as a vector **c** that is perpendicular to both **a** and **b**, with a direction given by the right-hand rule and a magnitude equal to the area of the parallelogram that the vectors span.



The cross product is defined by the formula

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin\theta \ \mathbf{n}$$

where ϑ is the angle between **a** and **b** in the plane containing them (hence, it is between 0° and 180°),

a and **b** are the magnitudes of vectors **a** and **b**,

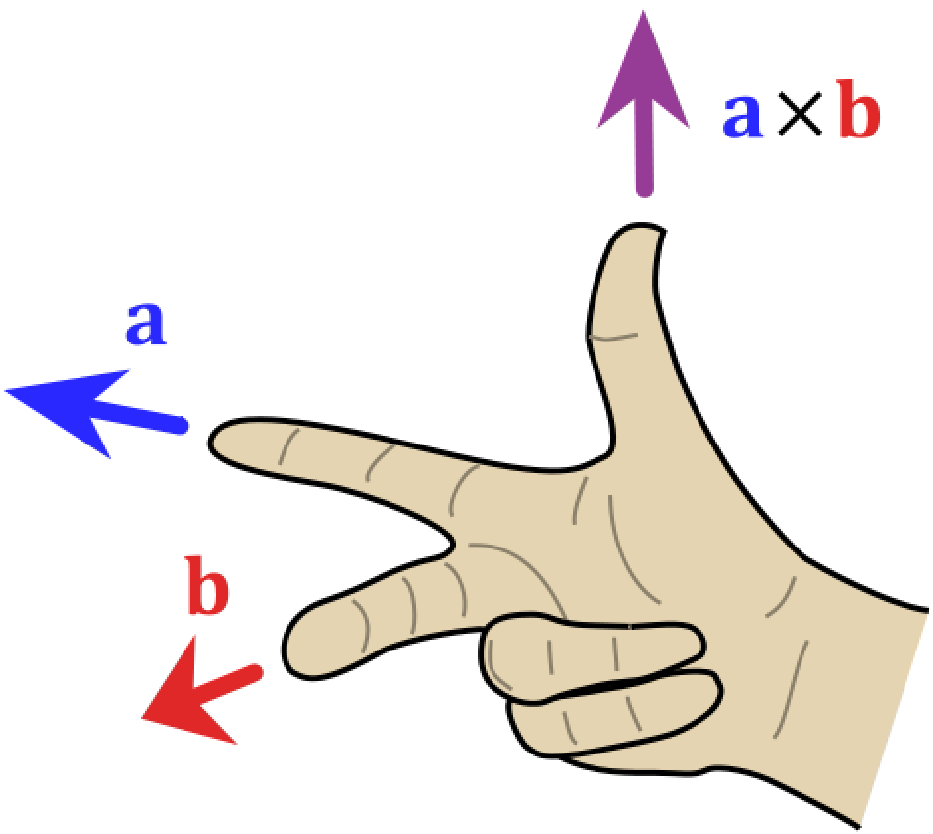
and **n** is a unit vector perpendicular to the plane containing **a** and **b** in the direction given by the right-hand rule (illustrated).

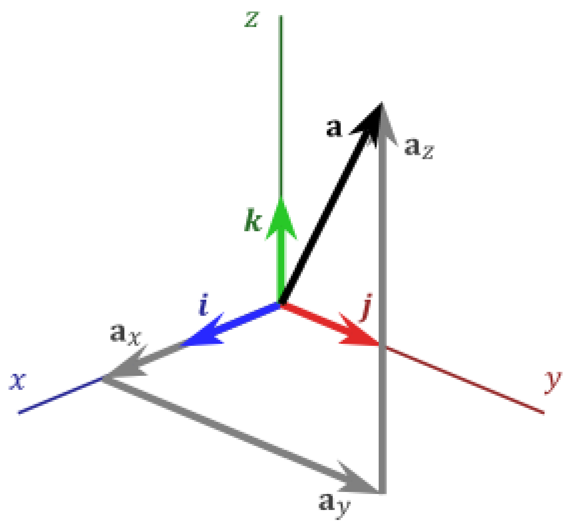
If the vectors **a** and **b** are parallel (i.e., the angle ϑ between them is either 0° or 180°), by the above formula, the cross product of **a** and **b** is the zero vector **0**.

By convention, the direction of the vector **n** is given by the right-hand rule, where one simply points the forefinger of the right hand in the direction of **a** and the middle finger in the direction of **b**. Then, the vector **n** is coming out of the thumb

Using this rule implies that the cross-product is anti-commutative, i.e., **b** × **a** = −(**a** × **b**).

Pointing the forefinger toward **b** first, and then pointing the middle finger toward **a**, the thumb will be forced in the opposite direction, reversing the sign of the product vector.





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The standard basis vectors **i**, **j**, and **k** satisfy the following equalities:

$$\mathbf{i} = \mathbf{j} \times \mathbf{k}$$
$$\mathbf{j} = \mathbf{k} \times \mathbf{i}$$
$$\mathbf{k} = \mathbf{i} \times \mathbf{j}$$

which imply, by the anticommutativity of the cross product, that

$$\mathbf{k} \times \mathbf{j} = -\mathbf{i}$$
$$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$$
$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$$

The definition of the cross product also implies that

$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$ (the zero vector).

$$\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$$
$$\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$$

Their cross product **u** × **v** can be expanded using distributivity:

$$\begin{aligned}\mathbf{u} \times \mathbf{v} = &(u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}) \times (v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}) \\ = &u_1v_1(\mathbf{i} \times \mathbf{i}) + u_1v_2(\mathbf{i} \times \mathbf{j}) + u_1v_3(\mathbf{i} \times \mathbf{k}) \\ &+ u_2v_1(\mathbf{j} \times \mathbf{i}) + u_2v_2(\mathbf{j} \times \mathbf{j}) + u_2v_3(\mathbf{j} \times \mathbf{k}) \\ &+ u_3v_1(\mathbf{k} \times \mathbf{i}) + u_3v_2(\mathbf{k} \times \mathbf{j}) + u_3v_3(\mathbf{k} \times \mathbf{k})\end{aligned}$$

$$\begin{aligned}\mathbf{u}\times\mathbf{v} =& u_1v_1\mathbf{0} + u_1v_2\mathbf{k} - u_1v_3\mathbf{j} \\ & - u_2v_1\mathbf{k} - u_2v_2\mathbf{0} + u_2v_3\mathbf{i} \\ & + u_3v_1\mathbf{j} - u_3v_2\mathbf{i} - u_3v_3\mathbf{0} \\ =& (u_2v_3 - u_3v_2)\mathbf{i} + (u_3v_1 - u_1v_3)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}.\end{aligned}$$

$$\begin{aligned}s_1 &= u_2v_3 - u_3v_2 \\ s_2 &= u_3v_1 - u_1v_3 \\ s_3 &= u_1v_2 - u_2v_1\end{aligned}$$

$$\mathbf{u}\times\mathbf{v} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}\mathbf{k}$$

The magnitude of the cross product can be interpreted as the positive area of the parallelogram having **a** and **b** as sides

$$A = \|\mathbf{a}\times\mathbf{b}\| = \|\mathbf{a}\|\,\|\mathbf{b}\|\sin\theta.$$

PROPERTIES OF CROSS PRODUCT:

- If the cross product of two vectors is the zero vector, (**a** × **b** = **0**), then either of them is the zero vector, (**a** = **0**, or **b** = **0**) or both of them are zero vectors, (**a** = **b** = **0**), or else they are parallel or antiparallel, (**a** || **b**), so that the sine of the angle between them is zero, (ϑ = 0° or ϑ = 180° and sinϑ = 0).

- The self cross product of a vector is the zero vector, i.e., **a** × **a** = **0**.

- The cross product is anticommutative,

$$\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a},$$

- distributive over addition,

$$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = (\mathbf{a}\times\mathbf{b}) + (\mathbf{a}\times\mathbf{c}),$$

- and compatible with scalar multiplication so that

$$(r\mathbf{a})\times\mathbf{b} = \mathbf{a}\times(r\mathbf{b}) = r(\mathbf{a}\times\mathbf{b}).$$

UNIT-2

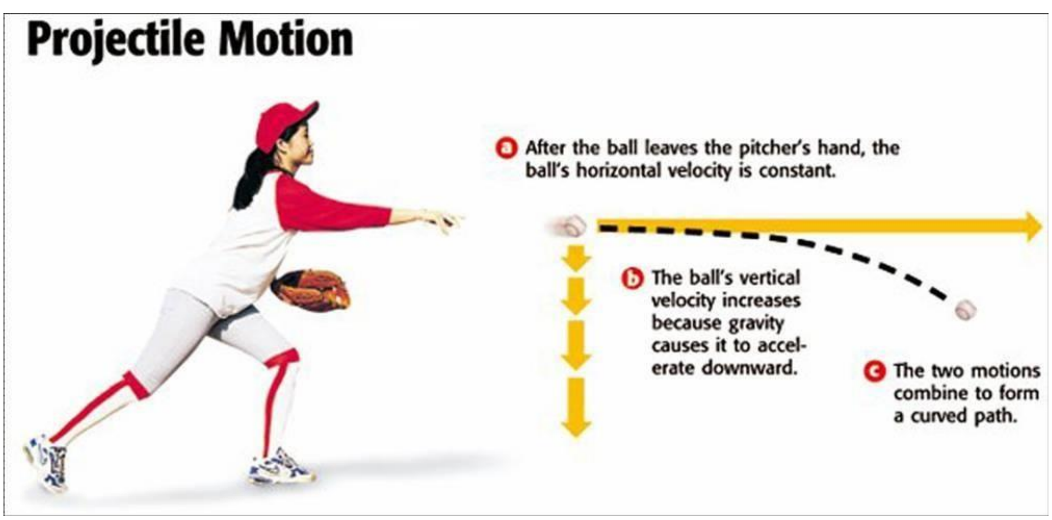
CURVILINEAR MOTION & KINEMATICS

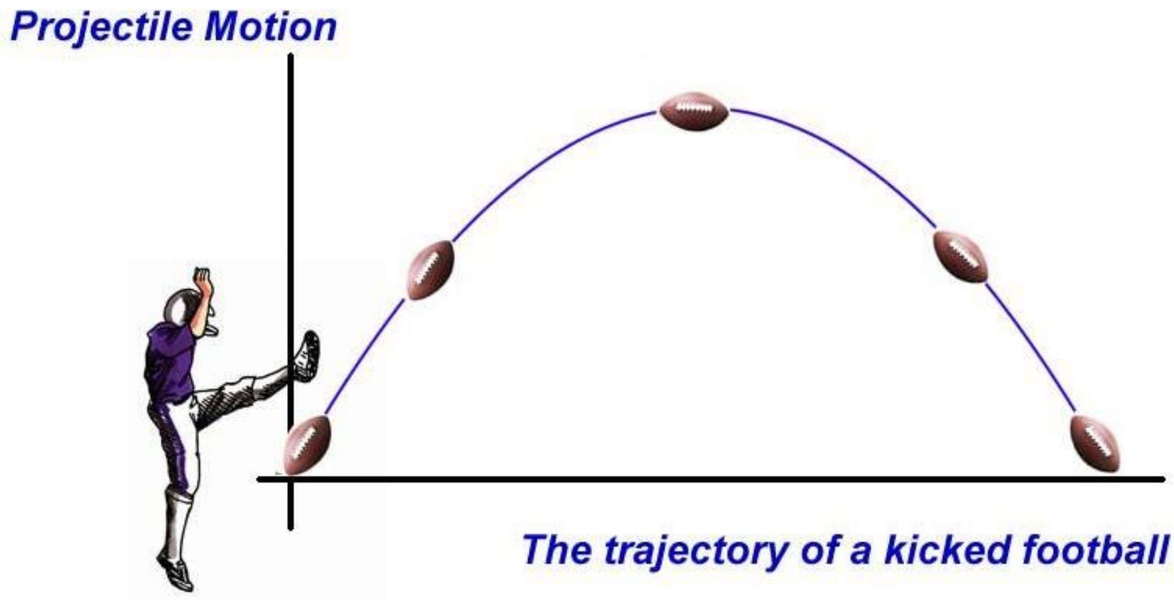
<u>PROJECTILE MOTION</u> :-

Definition & Concept:-

- A body projected into space and is no longer provided with any fuel is called projectile and motion of the body is said to be projectile motion.
- A projectile can be thrown into all possible directions into the 3D space, which is categorized into three parts, 1. along vertical direction. 2. along horizontal direction. along any direction making an angle of θ with horizontal.
- When the projectile is thrown towards the gravitational force of earth , acceleration of the object is equal the value of acceleration due to gravity. When it is thrown opposite to force of gravity, acceleration of the object is negative of the value of acceleration due to gravity.
- When the projectile is thrown in any direction making an angle θ with the horizontal, its motion can be consider as the resultant of horizontal and vertical motion.

Examples :-





Projectile is a body thrown with an initial velocity in the vertical plane and then it moves in two dimensions under the action of gravity alone without being propelled by any engine or fuel. Its motion is called projectile motion. The path of a projectile is called its trajectory.

Examples:

1. A packet released from an airplane in flight.
2. A golf ball in flight.
3. A bullet fired from a rifle.
4. A jet of water from a hole near the bottom of a water tank.

A body can be projected in three ways :

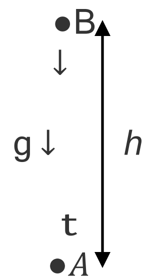
i. **Vertical Projection** – When the body is given an initial velocity in the Vertical direction only.
ii. **Horizontal projection**-When the body is given an initial velocity in the horizontal direction only.
iii. **Angular projection-**When the body is thrown with an initial velocity at an angle to the horizontal direction.

Let us consider all the three cases separately neglecting the effect of air resistance.

Let us take x-axis along the horizontal direction and y-axis along the vertical direction.

Case 1- **Vertical Projection :**

A body is thrown from point A with an initial velocity u along the vertical direction. Due to the action of acceleration due to gravity acting downwards, the velocity decreases and becomes zero at B.



Along x-axis	Along y-axis
1. Component of initial velocity along x-axis. **$u_x=0$**	1. Component of initial velocity along y-axis. **$u_y=u$**
2. Acceleration along x-axis **$a_x=0$** (Because no force is acting along the horizontal direction)	2.Acceleration along y-axis **$a_y=g=9.8m/s^2$** It is directed downwards.
3. Component of velocity along the x-axis at any instant **t**. **$v_x=0$**	3. Component of velocity along the y-axis at any instant **t.** **$v_y=u_y + a_yt$** **$=u + gt$** **$v_y= u + gt$**
4. The displacement along x-axis at any instant **t** **$x=u_xt + (1/2)\ a_xt^2$** **$x=0$**	4. The displacement along y-axis at any instant **t** **$y= u_yt + (1/2)\ a_yt^2$** **$y= ut - (1/2)\ gt^2$**

<u>velocity at any instant of time t</u>

We know ,at any instant **t**

$v_x= 0$

$v_y= u + gt$

$v= (v_x^2 + v_y^2)^{1/2} = u + gt$

Time of flight (T):
It is the total time for which the projectile is in flight (from A to B and back to A in the diagram above)

To find T we will find the time of ascent and descent

Time of ascent:-

v_y= u + gt

At B, **v_y = 0, g = -g (as the body is going upward)**

0= u - gt

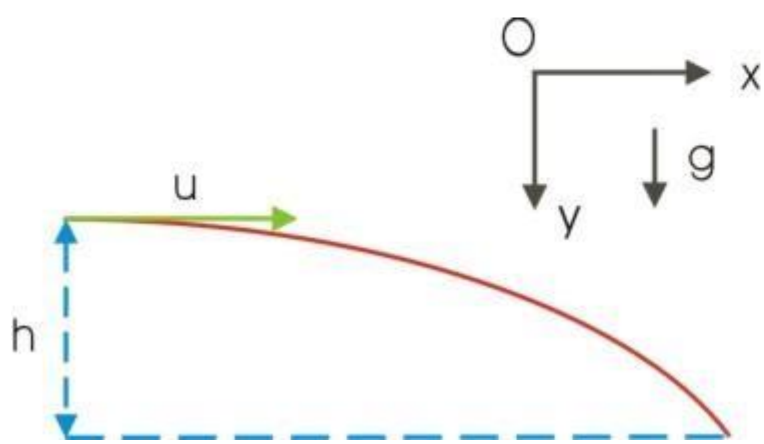
t = u/g

Hence, the time of flight = T= time of ascent+ time of descent= 2t= 2 u/g

Range (R) :
It is the horizontal distance covered during the time of flight T.
Since, **u_x=0, the** horizontal distance covered during the time of flight T =0

Case 2-- **Horizontlal Projection**

A body is thrown with an initial velocity u along the horizontal direction.
The motion along x and y axis will be considered separately.
Let us take the starting point to be at the origin.



Along x-axis	**Along y-axis**
1. Component of initial velocity along x-axis. $u_x=u$	1. Component of initial velocity along y-axis. $u_y=0$
2. Acceleration along x-axis $a_x=0$ (Because no force is acting along the horizontal direction)	2.Acceleration along y-axis $a_y=g=9.8m/s^2$ It is directed downwards.
3. Component of velocity along the x-axis at any instant **t**. $v_x=u_x + a_x t$ $=u + 0$ $v_x=u$ This means that the horizontal component of velocity does not change throughout the projectile motion.	3. Component of velocity along the y-axis at any instant **t.** $v_y=u_y + a_y t$ $=0 + gt$ $v_y=gt$
4. The displacement along x-axis at any instant **t** $x=u_x t + (1/2)\, a_x t^2$ $x=u_x t + 0$ $x=u\,t$	4. The displacement along y-axis at any instant **t** $y= u_y t + (1/2)\, a_y t^2$ $y= 0 + (1/2)\, a_y t^2$ $y=(1/2)\, gt^2$

Equation of a trajectory (path of a projectile)

We know, $x = ut$

$t=x/u$

Also, $y= (1/2)gt^2$

Subsituting for **t** we get

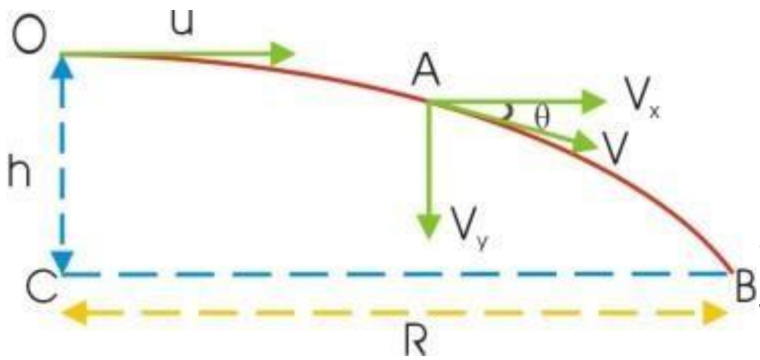
$y= (1/2)g(x/u)^2$

$y= (1/2)(g/u^2)x^2$

$y= kx^2$ where $k= g/(2u^2)$

This is the equation of a parabola which is symmetric about the y-axis. Thus, the path of projectile ,projected horizontally from a height above the ground is a parabola.

velocity at any instant of time t



We know ,at any instant **t**

$v_x = u$

$v_y = gt$

$$v = (v_x^2 + v_y^2)^{1/2} = [u^2 + (gt)^2]^{1/2}$$

Time of flight (T):

It is the total time for which the projectile is in flight (from O to B in the diagram above)

To find T we will find the time for vertical fall

From $y = u_y t + (1/2) gt^2$

At the point O, $y = h$, $t=T$

$h = 0 + (1/2) gt^2$

$T = (2h/g)^{1/2}$

Range (R) :

It is the horizontal distance covered during the time of flight T.

From $x = ut$

When $t=T$, $x=R$

$R=uT$

$R=u(2h/g)^{1/2}$

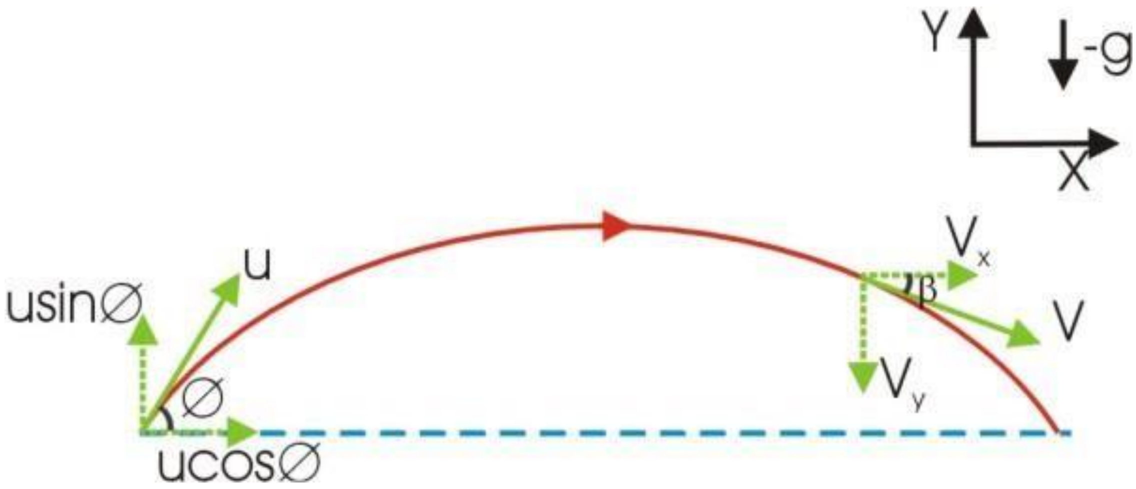
Case 3: Angular Projection:-

Let us consider the case when the object is projected with an initial velocity u at an angle Ø to the horizontal direction.

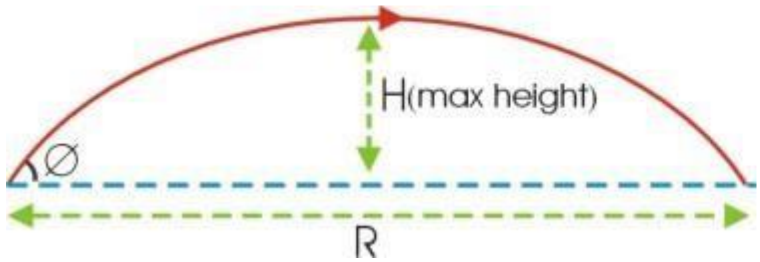
Let air resistance is negligible .

Since the body first goes up and then comes down after reaching the highest point , we will use the Cartesian convention for signs of different physical quantities.**The acceleration due to gravity 'g' will be negative as it acts downwards**.

Here the motion of body can be separated into horizontal motion (motion along x-axis) and vertical motion (motion along y-axis) .



X axis	Y axis
1. Component of initial velocity along x-axis. **$u_x = u\cos\Phi$**	1. Component of initial velocity along y-axis. **$u_y = u\sin\Phi$**
2. Acceleration along x-axis **$a_x = 0$** (Because no force is acting along the horizontal direction)	2.Acceleration along y-axis **$a_y = -g = -9.8 m/s^2$** (g is negative as it is acting in the downward direction)
3. Component of velocity along the x-axis at any instant **t**. **$v_x = u_x + a_x t$** **$= u\cos\Phi + 0 = u\cos\Phi$** **$v_x = u\cos\Phi$** This means that the horizontal component of velocity does not change throughout the projectile motion.	3. Component of velocity along the y-axis at any instant **t.** **$v_y = u_y + a_y t$** **$v_y = u\sin\Phi - gt$**
4. The displacement along x-axis at any instant**t** **$x = u_x t + (1/2)\, a_x t^2$** **$x = u\cos\Phi . t$**	4. The displacement along y-axis at any instant **t** **$y = u_y t + (1/2)\, a_y t^2$** **$y = u\sin\Phi . t - (1/2) g t^2$**



Equation of Trajectory (Path of projectile)

At any instant **t**

$x = u\cos\Phi . t$

$t = x/(u\cos\Phi)$

Also , **$y = u\sin\Phi . t - (1/2)gt^2$**

Substituting for **t**

$y = u\sin\Phi . x/(u\cos\Phi) - (1/2)g*x/(u\cos\Phi)+^2$

$y = x.\tan\Phi - [(1/2)g.\sec^2.x^2]/u^2$

This equation is of the form **$y = ax + bx^2$** where 'a' and 'b are constants. This is the equation of a parabola. Thus, the path of a projectile is a parabola .

velocity of the body at any instant of time t

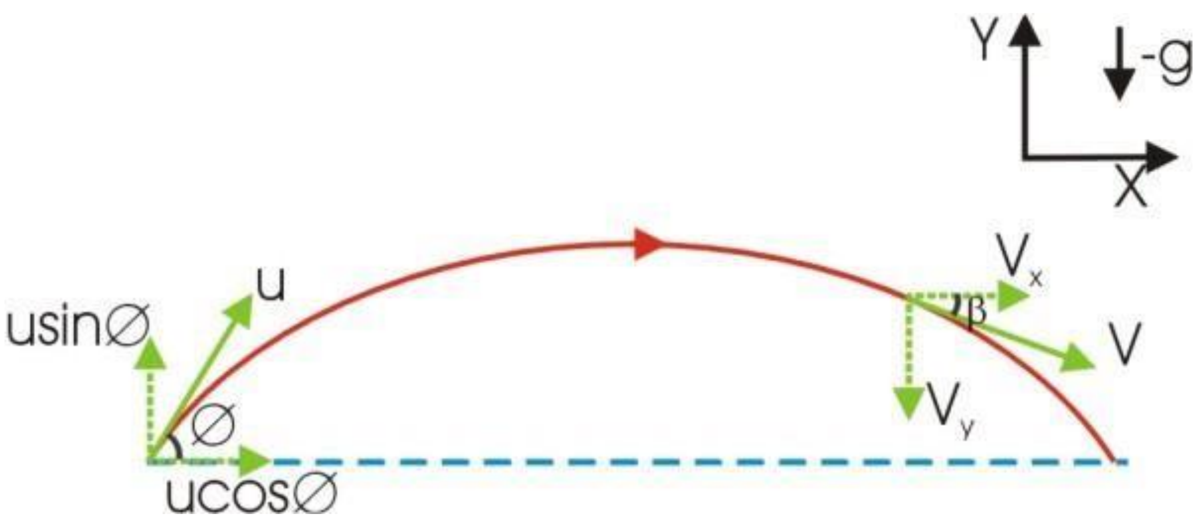
$v_x = u\cos\Phi$

$v_y = u\sin\Phi - gt$

$v = (v_x^2 + v_y^2)^{1/2}$

$= (u^2\cos^2\Phi + u^2\sin^2\Phi - 2u\sin\Phi gt + g^2t^2)$

$= u^2 - 2u\sin\Phi gt + g^2t^2$



Time of flight T

It is the time taken by the projectile to come back to the same level from which it was projected .i.e. It is the sum of time of ascent (rise) and time of descent (fall).

Angular Projectile motion is symmetrical about the highest point.

Hence, the time of ascent (rise) = time of descent (fall).

Time of ascent :-

Let, t= the time taken by the projectile to reach the highest point

.At the highest point,the vertical component of velocity $v_y = 0$

Applying the formula, $v_y = u\sin\Phi - gt$

$0 = u\sin\Phi - gt \implies t = u\sin\Phi/g$

Hence, $T = 2t = 2u\sin\Phi/g$

Maximum height H

Equation for vertical distance (y component)

$y = u_y t - (1/2)gt^2$

At , $t = T/2$, $y = H$

$H = u\sin\Phi.T/2 - (1/2)g(T/2)^2$

substituting **T**

$H = u\sin\Phi.u\sin\Phi/g - (1/2)\,g\,(u\sin\Phi/g)^2$

$= (u^2\sin^2\Phi)/g - (u^2\sin^2\Phi)/2g$

$H = (u^2\sin^2\Phi)/2g$

Range R

Range is the total horizontal distance covered during the time of flight.

From equation for horizontal motion, $x = u_x t$

When $t = T$, $x = R$

$R = u_x T = u\cos\Phi.2u\sin\Phi/g$

$= u^2 2\sin\Phi\cos\Phi/g = u^2\sin 2\Phi/g$ using, $2\sin\Phi\cos\Phi = \sin 2\Phi$,

$R = (u^2\sin 2\Phi)/g$

Condition for maximum horizontal range :-

The horizontal range depends upon the velocity of projection u and the angle of projection Φ**. For a given value of u, the range will be maximum when** ***sin2Φ*** will be maximum.

Hence, $\sin 2\Phi = 1 \implies \Phi = 45^0$, this is the condition for maximum range.

$R_{max} = u^2/g$

FRICTION-

Definition :- Frictional force is a force that resists movement between two objects. If friction is limiting, it is providing the maximum possible force it can.

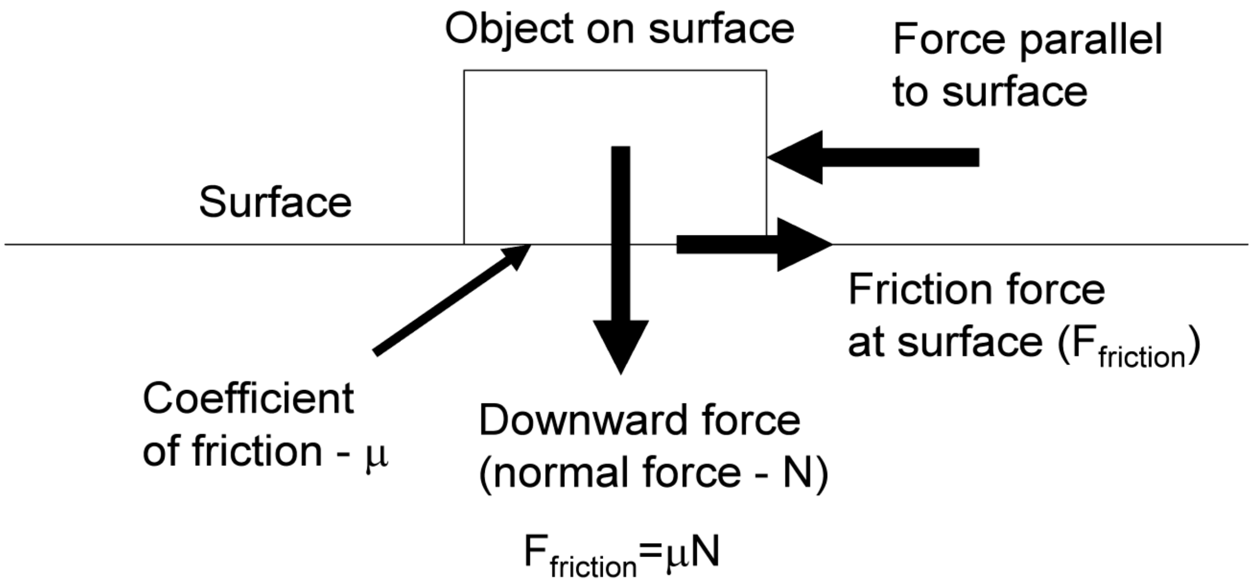
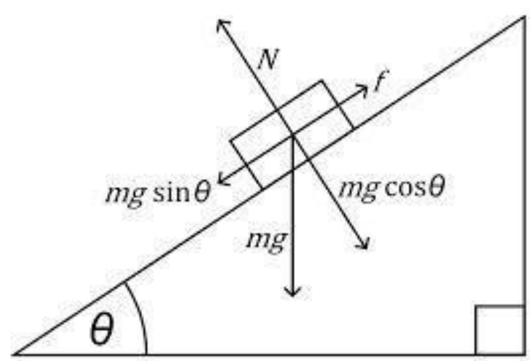


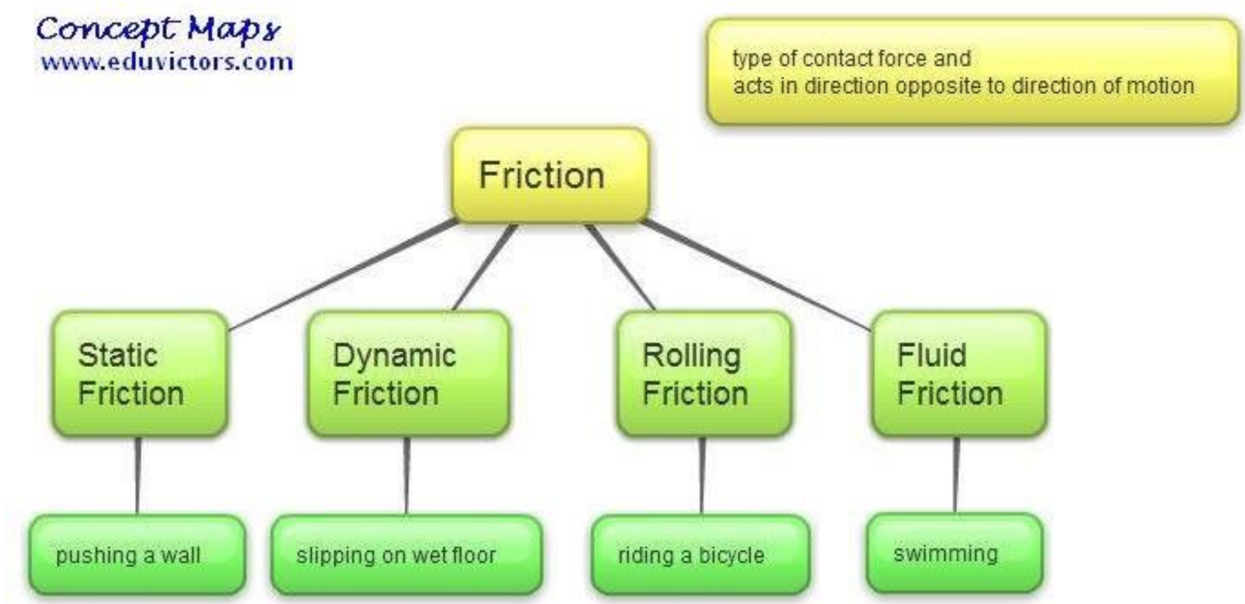
Figure 1 – Basic Definitions of the Coefficient of Friction



Types of Friction :-

There are four types of friction namely

1. Static friction
2. Kinetic friction
3. Rolling friction
4. Fluid friction



Static Friction:-

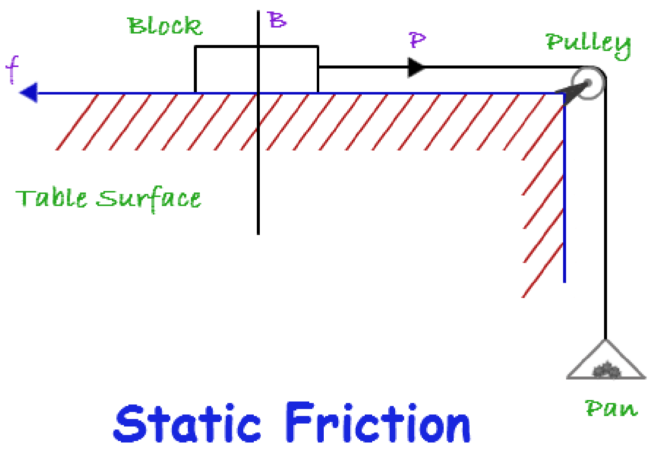
The resistance encountered by a body in static condition while tending to move under the action of an external force is called static friction (f). Static friction is equal and opposite to the applied force.

Static friction comes into play when a body is forced to move along a surface but movement does not start. The magnitude of static friction remains equal to the applied external force and the direction is always opposite to the direction of motion. The magnitude of static friction depends upon μ_s (coefficient of static friction) and N (net normal reaction of the body).

Example:- Consider a block 'B' which is resting on a horizontal table. Let a small pan be attached to the block by means of a horizontal thread passing over a smooth frictionless pulley. Initially when weight in the pan is zero, the body does not move because the applied force due to the weight in the pan is zero, the body does not move because the applied force due to the weight of the pan becomes equal and opposite to the force of friction between the table and the body. When the weight in the pan is increased the body may still be static. The body does not move because the resultant force on the body is zero. The frictional force is equal in magnitude and opposite in direction to the applied force 'P' and is tangential between the two surfaces.

When the applied force (P) is increased the frictional force also increases equally until the body starts moving. When it is about to slide on the table, the static friction reaches a maximum value. Any further increase in the applied force makes the body slide on the table.

The maximum value of static friction is called Limiting friction.



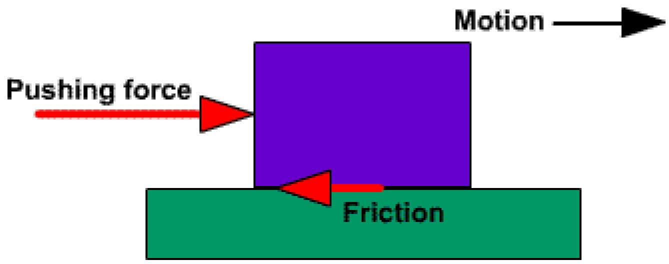
Dynamic or Kinetic Friction

The resistance encountered by a sliding body on a surface is known as kinetic friction or dynamic friction or sliding friction

Kinetic friction denoted as μk comes into play when a body just starts moving along a surface. When external applied force is sufficient to move a body along a surface then the force which opposes this motion is called as kinetic frictional force.

Magnitude of kinetic frictional force $f_k = \mu k$ N

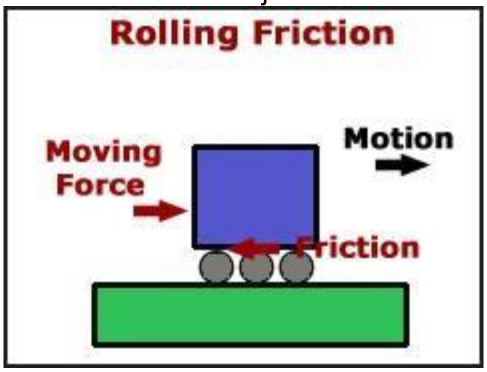
Where μk is coefficient of kinetic frictional force and N is the net normal reaction on the body. The magnitude of kinetic frictional force is always less than magnitude of static frictional force. When value of applied net external force F is more than f_k then body moves with a net acceleration and when these forces are equal then body moves with a constant velocity.



Rolling Friction

If a wheel or a cylinder or a spherical body like a marble rolls on a horizontal surface, the speed of rolling gradually decreases and it finally stops. The resistance encountered by a rolling body on the surface is known as Rolling friction

Rolling frictional force is a force that slows down the motion of a rolling object. Basically it is a combination of various types of frictional forces at point of contact of wheel and ground or surface. When a hard object moves along a hard surface then static and molecular friction force retards its motion. When soft object moves over a hard surface then its distortion makes it slow down.



Fluid Friction

When a body moves in a fluid or in air then there exists a resistive force which slows down the motion of the body, known as fluid frictional force.

A freely falling skydiver feels a drag force due to air which acts in the upward direction or in a direction opposite to skydiver’s motion. The magnitude of this drag force increases with increment in the downward velocity of skydiver. At a particular point of time the value of this drag force becomes equal to the driving force and skydiver falls with a constant velocity.

FORCE OF LIMITING FRICTION :-

when a horizontal force is applied to a static body to move the same, a frictional force equal to the applied force develops in the opposite direction resisting the motion. As long as the body does not move , this force is called static frictional force. Now if the applied force is increased, the frictional force in the opposite direction increases proportionately until it reaches the limit after which if the applied force is increased, the body starts moving. This threshold force is called static or limiting force of friction.

LAWS OF LIMITING FRICTION

- The direction of force of friction is always opposite to the direction of motion.
- Force of friction depends upon the nature and state of polish of the surfaces in contact.
- It acts tangentially to the interface between the two surfaces.
- Magnitude of limiting friction “F” is directly proportional to the normal reaction “R” between the two surfaces in contact.

$$F \propto R$$

$$\Rightarrow \mathbf{F} = \mu R$$

$\mu =$ The proportionality constant called the COEFFICIENT OF FRICTION

- Magnitude of limiting friction between two surfaces is independent of area and shape of surfaces in contact so long as the normal reaction remains the same.

Methods to reduce Friction:-

Introduction:-

Friction is a necessary evil. In some situations, it plays a positive role whereas in others, it is not needed. The net efficiency of any machine depends on the amount of friction present in that machine, because a large part of the input energy of all machines is wasted in overcoming friction between its various parts, and thus the total output from the machine decreases.

There is thus a need to find ways to decrease friction in order to make machines more efficient and to increase their life time. These ways of decreasing friction can be applied in different parts of machines according to the function of that part, and thus a more efficient machine is constructed.

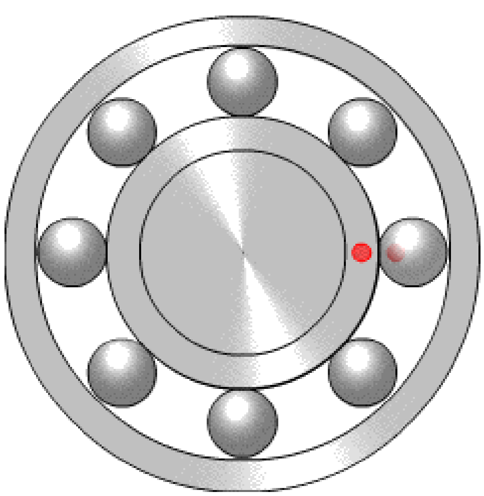
The following techniques are different ways to decrease friction used widely:

- **USE OF LUBRICANTS**

The use of lubricants like oil and grease helps to reduce friction by forming a thin film between different parts of a machine. This film covers up the scratches and lumps present on the surfaces of different parts and thus makes the surface more even than before. This reduces interlocking between the two surfaces, and hence the parts of the machine run smoothly.

- **USE OF BALL BEARINGS**

Two moving surfaces in a machine are fixed by placing small balls or rollers made out of steel in between them. This way, the moving parts avoid direct contact and sliding friction is changed to rolling friction. Rolling friction is less than sliding friction, thereby decreasing the amount of friction in the machine.



- **BY POLISHING**

The unevenness of surfaces can be reduced by polishing them. This will reduce the interlocking between surfaces in contact thus reducing friction.

- **USING SOFT, FINE POWDER**

Soft, fine powder like talcum powder and graphite powder also helps in filling in the microscopic scratches and grooves on a surface and thus make it more even and less prone to interlocking with other surfaces. Thus, friction is reduced.

- **STREAMLINING**

Streamlining is the process of making a machine’s shape such, that avoids resistance from air and water molecules while moving. For example airplanes have a streamlined shape which reduces friction between air molecules while flying. This is because fluids move with less friction over a streamlined surface, and the object moves forward as if cutting through them. The following diagram shows a streamlined and a blunt body:-

- **Use of correct combination of surfaces in contact:-**
  Use of alloys on moving and sliding parts reduces friction because alloys have a low coefficient of friction.